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Complexity in rainfall phenomena

R. Bove *, V. Pelino, L. De Leonibus

Servizio Meteorologico dell'Aeronautica, Centro Nazionale di Meteorologia e Climatologia Aeronautica, Aeroporto "De Bernardi", Via di Pratica di Mare, I-00040 Pratica di Mare, Roma, Italy

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Abstract

Recent studies demonstrate how rain is a self-organized critical phenomenon. Our findings about analysis of rain data collected from automatic stations (Data-Collection-Platforms) spread on the Italian territory confirm the results mentioned above. We show that power laws describe the number of rain events versus size and number of droughts versus duration. Anomalous Hurst coefficients and one-over-*f* (1/*f*) noise found are consistent with the concepts of criticality and self-similarity. This time structure of precipitation field implies that instantaneous sampling as that obtained by satellite remote sensing requires accomplishing short time scanning.

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1. Introduction

The knowledge of the effective precipitation on the ground is a fundamental information both for weather analysis and forecasting. The measure of precipitation has historically been on estimates of temporal averages by measuring the amount of water collected in a container after a certain time (typically hours or days), and recently by using Doppler radar collecting high-resolution

* Corresponding author. Tel.: +39 0691 293 214; fax: +39 0691 293 254.
E-mail address: bove@meteoam.it (R. Bove).

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data through indirect measurements. The focus of this paper is to investigate the statistical properties of rain and the applications in remote sensing techniques. The accumulated rain data we used have been collected with standard rain gauges from Data-Collection-Platforms (DCPs) with 15 min resolution. Data-Collection-Platform is the network of automatic and semi-automatic stations localized on all Italian territory in the number of about 60. In this work we introduce the concept of rain events as sequences of non-zero rain values, and we consider also the waiting times between successive events (droughts). Experiments reveal that rain is a complex dynamical system that, under certain conditions, will automatically adjust itself to a critical state characterized by power-law correlations in both space and time. This state is 'critical' in the sense of an equilibrium critical point where there is no characteristic length or time scale that controls the behaviour of the system. Rain is analogous to a variety of non-equilibrium relaxation processes in Nature such as earthquakes and avalanches. Many natural phenomena evolve intermittently, with periods of tranquillity interrupted by bursts of activity. The atmosphere is driven by a slow and constant energy input from the Sun. In particular, water is evaporated from the oceans. The energy is stored in the form of water vapour in the atmosphere. It is then suddenly released in bursts with no typical size, when the vapour condenses to water drops. A power law distribution of the number of rain events vs. size and number of droughts vs. duration was observed, displaying scale-free fluctuations.

Rainfall data reveal an anomalous Hurst exponent of the rain intensity signal, $H > 0.5$, indicating long-range correlations. The power spectra of rainfall time series exhibit $1/f$ noise over vastly different time scales illustrating that no typical time scale exists. These findings and the *emergence* of fractal structures indicate self-organized criticality (SOC) emerging naturally in interactive dissipative dynamical systems with many degrees of freedom. SOC provides a general mechanism for the emergence of complex behaviour in nature. The dynamics of a non-equilibrium system can become complex because has many components interacting nonlinearly and when the system is driven far from the equilibrium. Self-organized critical systems evolve toward a scale-free, or critical state naturally, without fine tuning any parameters. In the perspective of the complexity theory this approach can be useful to obtain new models of precipitation, particularly in drought and flooding hazard assessment. Because of temporal disomogeneity of the precipitation field, high temporal resolution and high sensitivity are needed for remote sensing measurements.

2. Power-laws and SOC in rain

The concept of self-organized criticality (SOC) was introduced in 1987 by Bak et al. [4]. This concept is well illustrated with a model of sandpile [18]. We consider a sandpile in which we add grains of sand for example, slowly and randomly. The pile initially is rather flat, with small local rearrangement or avalanches in time. As we add new grains, the pile grows more and more until the pile will reach a critical slope in a statistically stationary state. At this point, the addition of other grains may cause either small avalanches, or it may trigger a very large avalanche. In terms of energy the system is driven far from the equilibrium and tends to a uniform minimally stable state generated by some type of optimization process [15]. But this state is dramatically unstable exhibiting breakdown events of all sizes [13]. The states that emerge are those which are organized by the breakdown events. In a critical state we observe a power law spatial and temporal correlations in the distribution of breakdown events [5], like equilibrium systems undergoing a second order phase transition. Scale invariance behaviour or self-similarity has been observed in other natural phenomena such as earthquakes, landslides, fires and volcanic activity [3].

The distribution of energy released during an earthquake is a simple power law (Gutenburg–Richter law) [10]; biological evolution also exhibits long period of stasis punctuated by extinction events of all sizes [12]. Self-organized critical systems evolve toward a scale-free, or critical state naturally. The periods of stasis allow the system to remember its past, the intermittent events, in response to accumulated forcing over long time scales, make the system to evolve and become complex [2]. In the critical state there are very large correlations, so the individual degrees of freedom cannot be isolated. The interplay of many degrees of freedom, caused by co-evolutionary breakdown events, creates feedbacks that make the system highly non-linear and complex. Criticality, thus non-linearity and non-equilibrium, assures that even minor perturbations may have dramatic effects on the system. Defining rain events as sequences of non-zero rain intensities and drought periods, time intervals of zero rain, we find an analogy between the event-like structure of precipitations and the event-like structure of earthquakes or avalanches (Christensen et al., 2002). Our analysis on rainfall data reveals a power-law distribution for rain events vs. event size, as shown in Fig. 1.

Fig. 1 shows that there is no typical scale for rain intensity fluctuations but events of all sizes. Events interacting non-linearly arrange themselves in a fractal structure or self-similar, indicating SOC. Scaling region extends over several orders of magnitude following a simple power law:

$$N(M) \propto M^{-\tau_{\mathrm{M}}}, \quad \tau_{\mathrm{M}} \approx 2.35 \tag{1}$$

We can then calculate the number $N(M > M_1)$ of expected events exceeding a given mass M_1 [17]:

$$N(M > M_1) \propto \int_{M_1}^{\infty} M^{-\tau_{\mathrm{M}}}\,\mathrm{d}M = \frac{1}{\tau_{\mathrm{M}} - 1} M_1^{-\tau_{\mathrm{M}}+1} \tag{2}$$

A similar probability distribution of drought durations $N(T_{\mathrm{D}})$ was found to follow a power law

$$N(T_{\mathrm{D}}) \propto T_{\mathrm{D}}^{-\tau_{\mathrm{D}}}, \quad \tau_{\mathrm{D}} \approx 2.1 \tag{3}$$

This implies that scale invariance prevails also in drought distribution (Fig. 2).

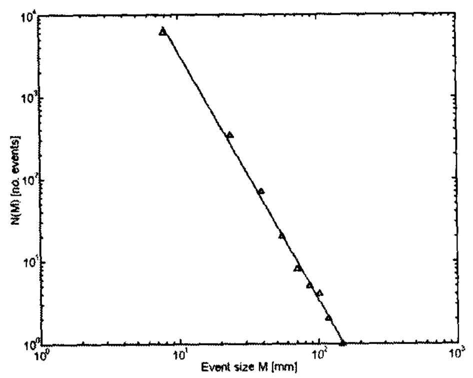

Fig. 1. Number density $N(M)$ of rain events vs. event size M (triangles) on a double logarithmic scale and fittin (straight line).

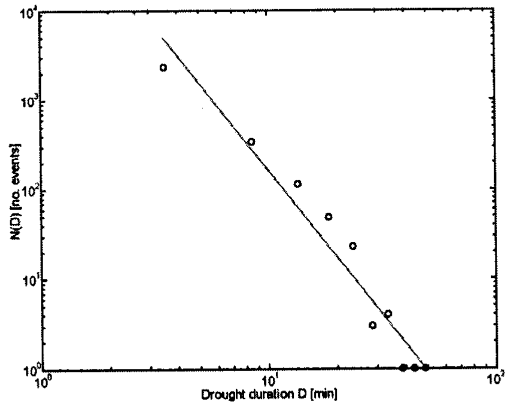

Fig. 2. Number density $N(D)$ of drought duration vs drought duration T_D (open circles) on a double logarithmic scale.

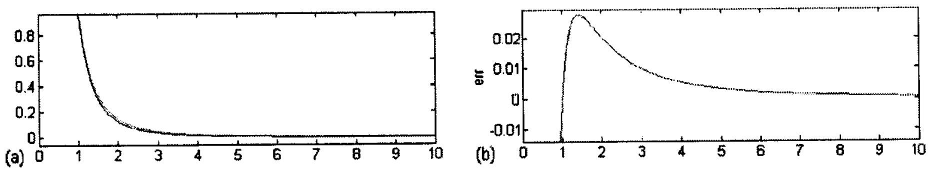

Fig. 3. Comparison between power laws obtained with exponents τ_M and τ_D in function of an adimensional parameter (a) and relative error (b).

This result could prove very useful in relation to drought hazard assessment or flooding hazard assessment [7,17]). For instance, in order to calculate the expected number $\overline{N}(T)$ of droughts with period $D > d$ in a given time period T, we have to integrate $N(D)$

$$\overline{N}(T) = T \cdot N(D > d) = T \cdot \frac{\text{const}}{2.1} \cdot d^{-2.1} \tag{4}$$

Moreover, it is important to highlight that there is non-trivial relationship between the duration and the event size during an event as shown in Fig. 3.

3. Hurst exponent

The Hurst exponent [11] occurs in several areas of applied mathematics, including fractals and chaos theory, long memory process and spectral analysis. Estimation of the Hurst exponent was originally developed in hydrology, but it occurs in several areas, in particular in Finance [1]. Estimating the Hurst exponent for a data set provides a measure of whether the data is a pure random walk or has underlying trends. Another way to state this is that a random process with an underlying trend has some degree of autocorrelation. When the autocorrelation has a very long (or mathematically infinite) decay this kind of Gaussian process (or random walk) is sometimes referred to as a *long memory process.*

Brownian walks can be generated from a defined Hurst exponent. If the Hurst exponent is $0.5 < H < 1.0$, the random walk will be a long memory process. Data set like this are sometimes referred to as fractional Brownian motion (fBm). Fractional Brownian motion is sometimes referred to as $1/f$ noise. The fractal dimension is directly related to the Hurst exponent for a statistically self-similar data-set ($D = 2 - H$). The values of the Hurst exponent range between 0 and 1. A value of 0.5 indicates a true random walk and there is no correlation between any element and a future element. A Hurst exponent value H, $0.5 < H < 1$ indicates persistent behaviour (positive autocorrelation): if there is an increase from time step t_{i-1} to t_i there will probably be an increase from t_i to t_{i+1}; a Hurst exponent value $0 < H < 0.5$ will indicate anti-persistent behaviour (negative autocorrelation): an increase will tend to be followed by a decrease. Hurst studied the problems related to water storage, in particular to the size of a reservoir construction that never runs out. His statistical method is called *rescaled range analysis* (R/S analysis).

This method consists in considering an imaginary reservoir in which the amount of water that flows into and out of it is a random process. In the case of inflow, the random process is driven by rainfall. In the case of outflow, the process is driven by demand for water. Using the data, the deviation from the average water level would be

$$X(t,\tau) = \sum_{u=0}^{t} (q(t) - \langle q \rangle_\tau) \tag{5}$$

where $\langle q(t) \rangle_\tau$ is an average inflow over time τ. The range $R(\tau)$, the difference between $X_{\max}$ and $X_{\min}$ of water level in the imaginary reservoir, over the period τ is

$$R(\tau) = \max_{1 \leqslant t \leqslant \tau} X(t,\tau) - \min_{1 \leqslant t \leqslant \tau} X(t,\tau) \tag{6}$$

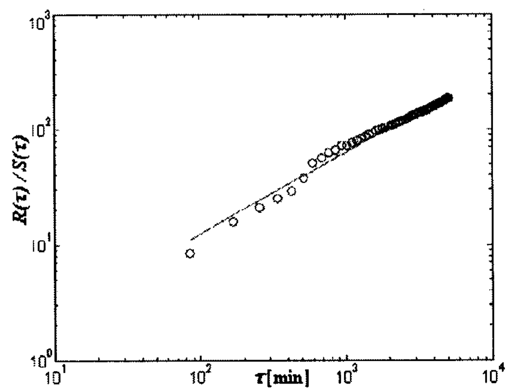

Fig. 4. The dimensionless ratio $R(\tau)/S(\tau)$ vs. τ (open circles) shown on a double logarithmic scale. The slope of the fitted straight line (solid) reveals the anomalous coefficient $H \approx 0.71$.

The rescaled range is calculated by dividing the range $R(\tau)$ by the standard deviation

$$S(\tau) = \sqrt{\frac{1}{\tau}\sum_{t=1}^{\tau}\{q(t) - \langle q\rangle_\tau\}^2} \tag{7}$$

The Hurst exponent is estimated by calculating the average rescaled range over multiple regions of the data. The expected value of R/S, calculated over a set of regions converges on the Hurst exponent power function, shown in Fig. 4.

The anomalous Hurst coefficient of $H \approx 0.71$ reveals long range correlations and SOC.

4. Spectral analysis

According to the Weiner–Khintchine theorem, the spectral density function can be expressed as the Fourier transform of the autocorrelation function for a random process [8]. The spectral density obtained from our data analysis on discrete rainfall time series is illustrated in Fig. 5. The power spectrum is averaged over several realizations (up to 60 DCP) to reduce the variance, which for one single realization is a 100% standard deviation.

The power spectrum displays a power-law behaviour $f^{-\beta}$ over vastly different time scales [20]. This kind of noise is named $1/f$ noise (or Flicker noise). It has been shown numerically that $1/f$ noise is a deterministic self-organized critical phenomenon emerging naturally in interactive dissipative dynamical systems with many degrees of freedom [6].

Critical states are barely stable and they are characterized by avalanches with power law spatial and temporal distribution functions. All interactions among the many components of the system are probabilistic, which is equivalent to introducing noise to the system. The level of noise is given by how strongly they interact. The noise propagates through the scaling clusters by means of a "domino" effect upsetting the minimally stable states. Breakdown events cause a cascade of energy dissipation on all length scales, such as in the turbulence phenomenon [4].

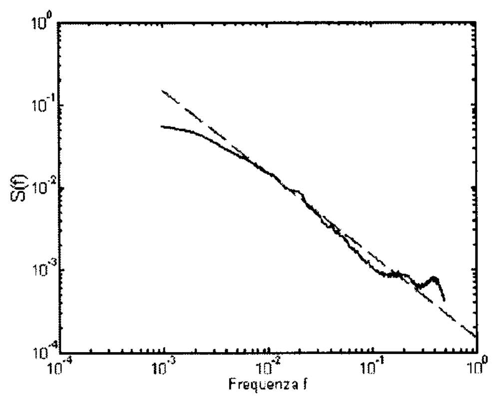

Fig. 5. Power spectrum of randomly superimposed rainfalls with log–log axis. The straight line is $1/f$ curve.

5. Conclusions

These results confirm that rain is an excellent example of a self-organized critical process. The statistics of the system resemble those of a closed system near the critical point of a phase transition. The findings suggest that rainfall time series cannot be reproduced by conventional methods of probability theory, but a better understanding of complexity in the atmosphere must be sought. Scale-free power-law behaviour is found to govern the statistics of rain over a wide range of time and event size scales. An anomalous Hurst exponent and $1/f$ noise reflect the dynamics of a self-organized critical state of minimally stable clusters of all length scales, which in turn generates fluctuations on all time scales.

This insight will inspire new research into the modelling of precipitation and atmospheric processes and might be useful in drought and flooding hazard assessment.

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